Diffusive Double Layer Model of PL Ring in Bacterial Flagellar Motor and Application to Nano-Machines

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New lubrication mechanism for nanomachine is proposed. This mechanism utilizes the effect of diffusiv e double layer observed in hydrophobic colloidal solution. Basic idea of the theory is inspired by the res earch for possible mechanism of bacterial flagellar motor. In this study, formulation of this mechanism is achieved and numerical calculation is performed. It is shown that this mechanism can produce enough loa d capacitance. Furthermore not only load capacitance to sustain driving force of flagellar motor

Keywords : Flagellar Motor, Diffusive Double Layer, Nano Lubrication, Tribology, Journal Bearing

1. INTRODUCTION

In the 20th century, tribology and lubrication engineering on the macroscale have reached a paradigm that most of the solid frictions in engineering can be well described by Coulomb-Amonton’s law and that the best strategy of lubrication is fluid film lubrication. Here, deviations from Coulomb’s law such as the friction of rubber or super-lubricity are considered to be exceptional cases.

Now, entering the 21st century, the circumstances around tribology are drastically changing. Since the paradigm is not always applicable to newly emerging areas such as microtribology, we have to employ some other paradigms for friction and lubrication.

In microtribology, friction between ideal crystalline surfaces may sometimes occur. Molecular forces would play an essential role in this kind of friction. Therefore, Coulomb’s law is not powerful anymore; as it is an experimental rule for solid friction. Molecular forces affect the behavior of liquids in molecularly thin liquid films. When thin film viscosity (or a viscosity rise) is induced by molecular forces, the fluid film lubrication method will not be the best anymore. Until now, we have not obtained enough knowledge about the lubrication in micromachines.

From the viewpoint of mechanical engineering, the bacterial flagellar motor has several interesting features in the flagellar motor of some bacterial species, there is a bushing, called the PL ring, which allows the rotating shaft to penetrate the rigid cell wall.

The existence of the PL ring suggests that our classical method of lubrication might be still valid in microtribology.

In a recent study, we estimated parameters in the tribology of the PL ring. By using theory of the classical hydrodynamic lubrication, we calculated load capacitance.[1]

Resulting load capacitance is order of 0.1pN(×10^-12) and this values is not adequate to compensate driving force (0.5pN). Furthermore, intermolecular force between rod and journal becomes order of 10 pN that is far bigger than driving force.

Thus, we concluded that new lubrication mechanism must be introduced to explain the behavior of PL ring. We proposed use of diffusive double layer as possible candidate of lubrication mechanism. We estimated the load capacitance generated by this effect and showed the adequate amount of repulsive force can be obtained by this mechanism.

In this study, we formulated lubrication theory by this mechanism and performed numerical calculation.

2. FORMULATION

The system can be described by Helmholtz’s free energy as

\[ F = \int kTn(\vec{r})[\ln(n(\vec{r})-1)]d\vec{r} \]

\[ + \int \epsilon[n(\vec{r})-n_{f}(\vec{r})][\phi(\vec{r})+\phi_{f}(\vec{r})]d\vec{r} \]

Here, \( n(\vec{r}) \) and \( n_{f}(\vec{r}) \) denote number density of mobile ion and fixed ion respectively.

Electric potentials caused by both ions are

\[ \phi = \frac{e}{4\pi\varepsilon_{0}} \int \frac{n(\vec{r})-1}{|\vec{r}'-\vec{r}|} d\vec{r}' \]

\[ \phi_{f} = \frac{e}{4\pi\varepsilon_{0}} \int \frac{n_{f}(\vec{r})-1}{|\vec{r}'-\vec{r}|} d\vec{r}' \]

The charge neutrality is always required as
\[ \int n(\vec{r}) d\vec{r} = \int n_0(\vec{r}) d\vec{r} \]  
(3)

Under these conditions, grand potential \( \Omega \)
\[ \Omega = F - \mu N = F - \mu \int n(\vec{r}) d\vec{r} \]  
(4)

is minimized with respect to \( n(\vec{r}) \). Resulting number density \( n_{eq}(\vec{r}) \) is called Poisson-Boltzman distribution as
\[ n_{eq}(\vec{r}) = \exp \left( -\frac{e\varphi_{eq}(\vec{r}) + e\varphi_f(\vec{r}) - \mu}{kT} \right) \]  
(5)

where \( n_{eq}(\vec{r}) \) and \( \varphi_{eq}(\vec{r}) \) are number density of mobile ion and electric potential caused by mobile ion in equilibrium respectively.

Load capacitance can be obtained as
\[ f = \left( \frac{\partial\varphi_{eq}}{\partial D} \right)_{\text{eq}, T} \]  
(6)

where D is distance of eccentricity of rod.

3. RESULTS AND DISCUSSION
Resulting distribution of mobile ions in various rod displacements are shown in Fig.1 (a)-(d). The load capacitance is calculated from numerical differentiation of eq.(6) and shown in Fig. 2. This result shows that the use of diffusive double layer gives enough load capacitance to sustain not only driving force but also the intermolecular force.

4. REFERENCES.